The

ITEA Journal

Answering the Question of Likelihood

Charles J. Ricke III

AVW Technologies, Inc., Chesapeake, Virginia

Published quarterly by the International Test and Evaluation Association

ITEA Journal 2013; 34: 230–234

Answering the Question of Likelihood

Charles J. Ricke III
AVW Technologies, Inc., Chesapeake, Virginia

Leaders must make decisions on how to apply scarce time and resources to the problems they face. Many times these decisions are based upon a risk assessment by supporting staffers. Subjective expected utility theory is broadly applied in the form of risk-reporting matrices as a "dashboard" for making rational decisions in the face of a range of consequences and likelihoods. The consequences of choices are, for the most part, definable in terms of cost, schedule, or performance. Likelihood determinations have not been as rigorous and are prone to subjective judgments. This article presents an objective method, based on a case study, for risk assessors to use in rationally and consistently determining the likelihood of a consequence and correctly portraying it in a risk-reporting matrix. Based upon this applied method, a general equation for a likelihood probability distribution function for risk-reporting matrices is proposed, followed by a discussion of its capabilities and limitations.

Key words: Risk management; likelihood; subjective expected utility; risk assessment; reporting matrix; risk exposure; probability distribution.

Risk managers determine where best to apply limited time and resources. These decisions are of the highest quality when they are rational and consistent. Given a wide range of options for expending time and resources, decisions are commonly based upon mitigating a perceived risk to the organization. The perceived risk is typically based upon two factors: the consequence of a risk (cost in money, time, or performance) and the potential (likelihood) for that risk to occur. Decision makers often use some sort of "dashboard" by which they can monitor the performance of their organization. The dashboard frequently chosen for risk management (OUSD [AT&L] 2006) is a risk-reporting matrix *(Figure 1)*. Risk managers can gauge the amount of exposure a risk presents using a rating system derived from the risk's placement in the cube (indicated in *Figure 1* by the multiple of consequence and likelihood). The risk-reporting matrix is an instantiation of subjective expected utility (SEU) theory (Savage 1954), where rational decisions are based upon both the cost (consequence) of a decision and the probability of that consequence occurring. A concise overview of Savage's SEU theory is provided by Karni (2005) and is not repeated here.

It has been this author's observation, through practical experience and review of risk-management literature (Office of the Undersecretary of Defense for Acquisition, Technology, and Logistics [OUSD(AT&L)] 2006; United States Government Accountability Office [USGAO] 2009; U.S. Department of Energy [DOE] 2011), that there is ample guidance for defining the consequences of a risk but less for determining the likelihood. Typical guidance on the placement of a risk in a cube is based upon a percentage, without a firm basis for how those percentages are determined. Likelihood can be hard to express in a linear risk scale when a large period of time is involved and risks present themselves with asymmetric periodicities and different failure rates. Consequently, a significant amount of room for subjective judgment in determining likelihood exists. The following sections present a case study in which a rational and consistent means for determining risk likelihood over the lifetime of a project is proposed. Following the case study, a general equation for a probability distribution function is presented, along with a discussion of its capabilities and limitations.

Case study

Problem statement

A project is expected to have a 50-year life cycle. Exposure to risk during the performance of periodic tasks is monitored using a five-by-five risk-reporting matrix as the dashboard *(Figure 1)*. The consequence for particular outcomes has already been rationally defined on a five-tiered (t = 5) scale. Likelihood evaluation is based upon two characteristics: (a) how often the tasks are performed and (b) the probability that a failure will occur (P_f) when performing that task. A method must be constructed so that these charac-

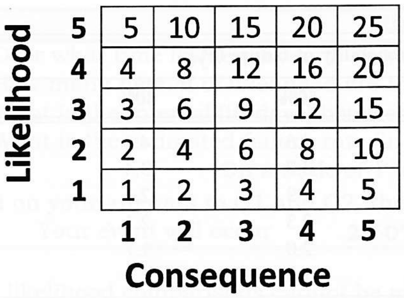

Figure 1. A typical risk-reporting matrix.

teristics can be logically and consistently applied to determine the likelihood of risks over the expected lifetime of the project.

Problem solution

SEU provides the amount of exposure each risk presents and provides a relative indication of how much leadership attention and resources should be applied to a given risk. SEU (DOE, 2011; Karni, 2005; Savage, 1954; USGAO, 2009) is given by the equation

$$\sum_{i=1}^{n} u(x_i) \cdot P(x_i),$$

where:

x_i = the possible outcome of an uncertain event;
$u(x_i)$ = the utility (i.e., consequence) of a given outcome; and
$P(x_i)$ = the subjective probability distribution function of each outcome.

$P(x_i)$ must be designed to logically and consistently expresses the likelihood of a given outcome. It must also be designed so that the resulting analysis fits within the five-tier risk-reporting matrix.

The first step in defining $P(x_i)$ is bounding the problem space. A 50-year life span equates to a range of 18,250 days of problem space (I). Using 1 day as the most frequently encountered instance of task performance, the project's expected lifetime can be divided into five orders of magnitude (m). Using the equation $m^T = I$ and substituting 5 for T and 18,250 for I, solving for m results in a five-tiered scale with an order of magnitude of 7.12 between tiers.

A logical next subdivision of the problem space (I) would be one that occurs one-seventh as often as the most frequent occurrence. For this case, that periodicity corresponds to 1 week (every 7 days). Other periodicities of task performance can be used to further subdivide the project's expected lifetime:

- 1 service lifetime,
- 50 annual tasks,
- 100 semiannual tasks,
- 200 quarterly tasks,
- 600 monthly tasks,
- 2,607 weekly tasks, and
- 18,250 daily tasks.

Using the equation $P(x_i) = \log_{7.12}(x_i)$, the relative likelihood of periodic events or tasks occurring over the 50-year life span of the project can be mapped to the five-tiered risk-reporting matrix *(Table 1)*.

Now that the solution space has been defined and scaled to determine the relative likelihood that a task is performed, the probability of observing a failure during an event or task—$P_f(x_i)$—is introduced. The approximate probability of the occurrence of a failure is expressed in percentages of 10, 30, 50, 70, and 90 percent.[1] Using the equation $P(x_i) = \log_{7.12}(x_i{}^*P_f(x_i))$, the calculated or estimated $P_f(x_i)$ modulates the relative likelihood derived previously to account for the likelihood that a failure will occur while performing a task. The result of that modulation is shown in *Table 2*.

It is worth noting that the logarithmic function is continuous and monotonic. Likelihood can be calculated to any desired degree of accuracy. A given consequence can now be assigned a discrete likelihood from 1 to 5 using a decision matrix:

Likelihood of 5 when $4.5 \le P(x_i) \le 5.0$.
Likelihood of 4 when $3.5 \le P(x_i) \le 4.5$.
Likelihood of 3 when $2.5 \le P(x_i) \le 3.5$.
Likelihood of 2 when $1.5 \le P(x_i) \le 2.5$.
Likelihood of 1 when $P(x_i) < 1.5$.

A lookup table can now be constructed to allow likelihood assessments to be made and reported on a scale of 1 to 5 without calculation *(Table 3)*. Interpolation is allowed because $P_f(x_i)$ is continuous. For example: If a problem occurs 50 percent of the time when a monthly task is performed, its likelihood would be 3 on a scale of 1 to 5. Similarly, if a problem occurs

Table 1. Periodic task performance of a 50-year project life scaled to five orders of magnitude.

Frequency of task	Daily	Weekly	Monthly	Quarterly	Semiannually	Annually	Lifetime
Outcomes (x_i)	18,250	2,607	600	200	100	50	1
$\log_{7.12}(x_i)$	5.0	4.0	3.3	2.7	2.3	2.0	0.0

Table 2. Scaled likelihood based upon task periodicity and P_f.

Frequency of event or task	x_i	$\log_{7.12}(x_i)$	Probability of failure (P_f) 90%	70%	50%	30%	10%
Daily	18,250	5.0	4.9	4.8	4.6	4.4	3.8
Weekly	2,607	4.0	4.0	3.8	3.7	3.4	2.8
Monthly	600	3.3	3.2	3.1	2.9	2.6	2.1
Quarterly	200	2.7	2.6	2.5	2.3	2.1	1.5
Semiannually	100	2.3	2.3	2.2	2.0	1.7	1.2
Annually	50	2.0	1.9	1.8	1.6	1.4	0.8

25 percent of the time during a daily task, its likelihood would be 4.

General equation for risk-reporting matrix likelihood scale

General equation

This case study was tailored to remove ambiguity in determining the likelihood of risks for a long-lived project that presented a problem space over several orders of magnitude. It was also designed to fit within the organization's five-tier risk-reporting matrix. The problem solution outlined can be generalized to allow broader application. The general equation for $P(x_i)$ is proposed:

$$P(x_i) = \log_m(x_i \cdot P_f(x_i)),$$

where:

- $m = \sqrt[T]{I}$ = the order of magnitude between each likelihood tier and the next;
- T = the number of decision tiers (i.e., scale);
- I = the highest number of event or task repetitions that occur within the time frame examined;
- x_i = the number of times an event or task occurs within the time frame examined; and
- $P_f(x_i)$ = the probability of a failure during an event or task.

Because $P(x_i)$ is continuous, likelihood can be calculated and scaled over user-defined time frames and task periodicities *(Figure 2)*. User definition of both the problem space and the number of decision tiers allows this method to be applied to a wide range of situations. For the earlier case study, a task performed (or anticipated to be performed) every 7 days that has (or is anticipated to have) a 25 percent failure rate would have a corresponding likelihood of 3.3 on a five-tiered risk-reporting matrix. The same risk on a seven-tiered risk-reporting matrix would have a likelihood of 4.6.

Capabilities and limitations

The proposed $P(x_i)$ uses classic probability-determination methods for all likelihood calculations and is not new in that respect. Its unique feature is that it calculates and scales likelihood determinations from user-defined inputs on time frames, risk-reporting matrix scales, event and task intervals (periods), and actual or estimated failure rates (so long as the failure rate is not zero [very low failure rates are permitted]). $P(x_i)$ is also customizable to any user-defined failure-rate probability distribution function $P_f(x_i)$, which can be used to account for any number of factors (equipment failure, human failure, process failure, etc.). Inputs of time frame and period must also be converted to a common unit of measure for accurate results.

The likelihood of a nonrepeating event cannot be calculated. The proposed $P(x_i)$ is based upon the recurrence of an event or task in relation to the user-defined time frame (i.e., it has to occur more than once). This limitation is reflected in $P(x_i)$ when negative likelihood values are calculated (discussed in

Table 3. Example likelihood lookup table.

Frequency of event or task	Probability of failure (P_f) 90%	70%	50%	30%	10%
Daily	5	5	5	4	4
Weekly	4	4	4	3	3
Monthly	3	3	3	3	2
Quarterly	3	3	2	2	2
Semiannually	2	2	2	2	1
Annually	2	2	2	1	1

Risk Likelihood Calculator	
Q1: Over what time frame (in years) do you want to evaluate the likelihood of a risk? →	50
Q2: How many tiers of of likelihood are there in your risk reporting matrix? →	5
Q3: What is the interval (in days) between occurrences of the event/task you are evaluating? →	7
Q4: What is the estimated failure rate (in %) of the event/task you are evaluating? →	25
On a scale of 1 to **5** the likelihood of your risk occurring is:	**3.3**
Based on your answers to Q1 and Q2, the likelihood scaling factor for your risk reporting matrix is:	7.12
Your event will occur **2,607** times over the time frame you selected in Q1.	

Notes:
1. Risk likelihood comparisons cannot be made unless they are observed over the same time period (Q1) and on the same scale (Q2).
2. The evaluation frame of reference is established by the response to Q1.
3. Likelihood is scaled to the number of decision tiers by the response to Q2.
4. The answer to Q4 cannot be zero. Extreemly low failure rates can be estimated.

Figure 2. Example Excel-based risk-reporting matrix likelihood calculator.

the next paragraph) for events that do repeat but not within the user-defined time frame.

$P(x_i)$ is capable of providing likelihood results outside the chosen time frame (scale) that require interpretation. Negative values of likelihood occur because the period of the event or task is greater than the chosen time frame or $P_f(x_i)$ is so low that a failure may occur only once within the chosen time frame. Even though a risk may be evaluated as having a low or negligible likelihood, this should not prevent risk assessors from reporting the risk so that managers can consider mitigation if its consequence is deemed catastrophic.

Risk management is predictive in nature because the first word in any risk statement is "if." Risk evaluators have to rationally explain their input choices to $P(x_i)$. Parametric data, research of similar events or processes, training and maintenance requirements, and user experience can be used to determine time frames, event or task periods, and $P_f(x_i)$.

Meaningful comparisons between relative likelihoods of periodic events can only be done so long as the time frame for their observation is the same. An event or task with a fixed periodicity occurs relatively more or fewer times as the time frame is increased or decreased. This is reflected in $P(x_i)$ outputs. The time frame for likelihood determination should be agreed upon by assessors and managers. Any changes to the time frame will require all relative likelihoods to be reassessed.

Implications for risk assessors and managers

Traditional risk likelihood has been expressed in percentages (DOE 2011, OUSD[AT&L] 2006) using various definitions. Risk assessors and managers are used to thinking of likelihood in these terms. The proposed methodology requires a small paradigm change. Using the proposed general equation, likelihood is expressed in orders of relative magnitude (m) vice percentages. In order for a risk to be lowered to the next tier (T), its likelihood must be reduced (by reductions in either $P_f(x_i)$ or task repetitions) by up to a factor of $1/m$. This paradigm shift has the potential, dependent upon the level of leadership scrutiny, to put additional pressure on management teams to justify their reasoning for reporting that the likelihood of a risk has been lowered.

Conclusion

Leaders face many problems to solve and have to decide where to apply precious time and resources to solve them. Which problem to solve first is often determined by how much exposure a risk presents to the organization. Risk, comprising both consequence and likelihood, should be rationally and consistently evaluated so that leadership can mitigate the highest risks first. The consequence of a risk is relatively easy to express compared to how likely it is to occur. Likelihood can be hard to express in a linear risk scale when a large period of time is involved and risks present themselves with asymmetric periodicities and different failure rates. The methodology used in the case study leads to a generalized equation for rationally determining risk likelihood. It can be used to provide leadership with consistent likelihood determinations in a risk-reporting matrix. Its scalability permits it to be used at any desired level of resolution. ❑

Charles J. (Joe) Ricke III, LCDR, USN (Ret), is a senior military analyst for AVW Technologies Inc., based in Chesapeake, Virginia. Mr. Ricke has over 12 years of test and evaluation experience on multiple defense acquisition programs. He holds a bachelor of science in electrical engineering from Southern Methodist University in Dallas, Texas, and a master of science in leadership from Grand Canyon University in Phoenix, Arizona. Mr.

Ricke is a member of the International Test and Evaluation Association (ITEA) and Alpha Chi. E-mail: ricke@avwtech.com

Endnotes

[1]It should be noted that a percentage scale (0 to 100 percent) could be expressed exponentially (0, 10^0, 10^1, and 10^2).

References

Karni, E. 2005. *Savage's subjective expected utility model*. Baltimore: Johns Hopkins University, http://www.econ2.jhu.edu/people/Karni/savageseu.pdf (accessed March 23, 2013).

Office of the Undersecretary of Defense for Acquisition, Technology and Logistics. 2006. *Risk management guide for DOD acquisition*. Washington, D.C.: Department of Defense.

Savage, Leonard J. 1954. *The foundations of statistics*. New York: Wiley.

United States Government Accountability Office. 2009. *GAO-09-3SP: Government Accountability Office cost estimating and assessment guide*. Washington, D. C.: Government Accountability Office.

U.S. Department of Energy. 2011. *DOE G 413.3-7A: Risk management guide*. Washington, D.C.: Department of Energy.

Acknowledgments

Thank you to Bob McIntyre, Ph.D., for his guidance and Curtis S. Eggers for his constructive feedback.